

Name \_\_\_\_\_ Test 1, Spring 2021

1) Find the product below. (15 points)

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 7 & 1 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & -2 \\ 0 & 2 & 5 \\ -1 & 0 & 3 \end{bmatrix}$$

2) Row reduce the matrix below to reduced echelon form. (15 points)

$$\begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 8 & 11 \\ 4 & 10 & 14 \end{bmatrix}$$

3) Find the null space of the matrix below. (15 points)

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

4) Express the span below in set builder notation. Do not include redundant vectors. (10 points)

$$\text{span} \left( \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -4 \end{bmatrix} \right\} \right)$$

5) Answer the following questions. (3 points each)

A) Let  $A$  be a  $4 \times 4$  matrix which, when row reduced, has 4 pivots. How many solutions can the equation  $A\vec{x} = \vec{0}$  have?

B) Let  $A$  be a  $4 \times 4$  matrix which, when row reduced, has 3 pivots. How many solutions can the

equation  $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$  have?

C) Let  $A\vec{x} = \vec{0}$  be a system of 5 equations in 3 variables. If the row space of  $A$  is  $\mathbb{R}^3$ , how many solutions can the system have?

D) Let  $A$  be a  $6 \times 7$  matrix for which  $A\vec{x} = \vec{b}$  with  $\vec{b} \neq \vec{0}$  has no solutions. When row reduced, what is the maximum number of pivots  $A$  can have?

E) Let  $A$  be a  $3 \times 3$  matrix that is a product of elementary matrices. Does  $A$  have an inverse?

6) Multiply the matrices below. (6 points)

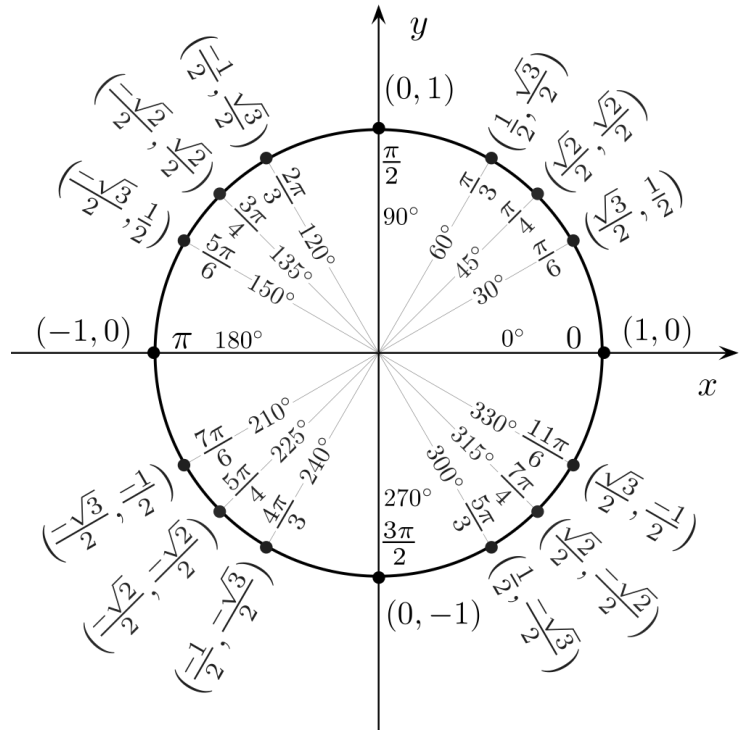
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & 0 & 0 \\ 2 & 3 & 4 & 5 & 3 \\ 6 & 7 & 8 & 9 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 5 & 6 & 7 & 8 & 5 \end{bmatrix}$$

7) Multiply the matrices below (6 points)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & 0 & 0 \\ 2 & 3 & 4 & 5 & 3 \\ 6 & 7 & 8 & 9 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 5 & 6 & 7 & 8 & 5 \end{bmatrix}$$

8) Find the angle between the two vectors below. You may use the unit circle provided here. (6 points)

$$\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \vec{w} = \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$



9) Solve the system of equations below. (6 points)

$$\begin{aligned}x_1 - x_3 &= 5 \\x_2 + 2x_3 &= 4\end{aligned}$$

10) How many solutions does matrix equation below have? (6 points)

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$